# CAE Working Paper #09-02

# The Mechanics of Central Bank Intervention in Foreign Exchange Markets

by

Kaushik Basu

January 2009

# The Mechanics of Central Bank Intervention in Foreign Exchange Markets

#### Kaushik Basu

Department of Economics Cornell University Ithaca, New York 14853 Email: kb40@cornell.edu

#### **Abstract**

Central banks in developing countries, wanting to devalue the domestic currency, usually intervene in the foreign exchange market by buying up foreign currency using domestic money—often backing this up with sterilization to counter inflationary pressures. Such interventions are usually effective in devaluing the currency but lead to a build up of foreign exchange reserves beyond what the central bank may need. The present paper analyzes the 'mechanics' of such central bank interventions and, using techniques of industrial organization theory, proposes new kinds of interventions which have the same desired effect on the exchange rate, without causing a build up of reserves.

**Key words.** Exchange rate, oligopoly theory, central bank intervention, foreign exchange dealers, India

**JEL classification numbers.** L31, D43, F31, G20

Acknowledgements. The work on this project began with a visit to the Reserve Bank of India, Mumbai, in August-September 2008, during which time I interviewed a large number of economists and officials at the Reserve Bank. I am grateful, in particular, to Y.V. Reddy, Rakesh Mohan, Shyamala Gopinath, Chandan Sinha and Michael Patra for valuable discussions. Subsequently, as the paper began to take shape, I have had the benefit of conversations with and comments from Talia Bar, Saugata Bhattacharya, Jonathan Cave, A. Chari, Sayantan Ghoshal, Annemie Maertens, Ted O'Donaghue, Ila Patnaik, Heraklis Polemarchakis, Eswar Prasad, and Assaf Razin. The paper also benefited greatly from a presentation at Warwick University.

# The Mechanics of Central Bank Intervention in Foreign Exchange Markets

### 1. Introduction

The paper addresses a practical policy problem faced by India's central bank, the Reserve Bank of India (RBI), and, to a certain extent, by all central banks of developing nations. The policy problem concerns the art of intervening in the foreign exchange market in order to influence the exchange rate in desirable directions. When, for instance, a nation's currency gets over-valued because of some shock and the central bank believes that this is a temporary fluctuation, it may want to devalue the currency in order to dampen the fluctuation. Moreover, some developing economies have followed the policy of deliberately keeping its currency a little under-valued so as to boost exports. Without going into the desirability of such interventions, I want to analyze the most efficient way to make such an intervention. In India, when the RBI wants to devalue the rupee it typically does so by using rupees to buy up foreign currency from the market. This strengthens the foreign currency and weakens the rupee. The details of exactly how the intervention is carried out are not public information. However, the broad outlines are easy to discern by reading publicly-available documents and from on-the-job experience and I describe these later in the paper.

The buying and selling of currency to influence the exchange rate is not special to India. People's Bank of China does this and so does the Fed in the US, though only on occasions. One fall-out of this kind of intervention to keep a nation's currency undervalued is that it leads to a build-up of foreign currency reserves beyond what a nation might need. The present paper draws on fairly standard industrial organization theory to argue that it is possible to design interventions in the foreign exchange market by central banks which have the desired effect of depreciating a nation's currency without resulting in a build up of foreign exchange reserves. While the present paper cannot make claims to theoretical novelty, its value lies, hopefully, in its ability to draw on theory from diverse sources and molding it to address this important policy problem.

It seems so axiomatic that buying a good is the way to boost the price of the good that most central banks give very little thought to the design of such interventions beyond deciding on *how much* foreign exchange to purchase. The new kind of intervention that is proposed in this paper depends critically on the structure of the market for foreign exchange that prevails in a nation. Hence, the main technical part of the paper is introduced by describing the market structure at length. The paper also develops a somewhat unusual characterization of bilateral oligopoly, which also allows me to generalize the nature of the result obtained in the paper. The last section is like a technical appendix that describes this model of bilateral oligopoly.

It is useful to begin by giving a general introduction to the debate on exchange rate management in India, which gives rise to the question that is the focus of this paper. This is done in the section that follows and should be viewed as a preamble for the modeling and theory that occurs thereafter.

# 2. The Background Story

All central banks, be it the Reserve Bank of India (RBI) the Fed in the U.S., the Bank of England or the People's Bank of China (PBC), keep a watch on the exchange rate of the nation, and when the rate veers too widely 'off course' these central banks try to intervene directly or indirectly in order to make 'corrections'.<sup>2</sup> It is widely believed that China's PBC tries to keep the renminbi undervalued, thereby making Chinese exports more attractive. On the other hand, the Indian Reserve Bank's ostensible aim is to dampen the fluctuations of the exchange rate. To quote from the RBI's most recent **Annual Report** (Reserve Bank of India, 2008a, p. 127): "India is classified under the 'managed float' exchange rate regime of the IMF. The Reserve Bank intervenes in the foreign exchange market to contain excessive volatility as and when necessary." However, given that foreign exchange reserves have risen steadily over the last decade

\_

<sup>&</sup>lt;sup>1</sup> My experience at the RBI certainly confirms this. While the amount of empirical research being conducted at the RBI is extremely impressive, there is not enough theoretical research. As I hope to demonstrate in this paper, the returns to such research are likely to be very high.

<sup>&</sup>lt;sup>2</sup> That exchange rates "matter" is standard wisdom in neoclassical economics as long as there is even one good that is non-traded (Flanders and Helpman, 1978). In more elaborate models of political economy governments can have a variety of reasons to influence exchange rates even if all goods were traded.

and a half and now stands well above 300 billion dollars, it is arguable that India also has a policy of keeping the rupee undervalued, although not as aggressively as China. In 2007-08 the Reserve Bank made net purchases of 78.2 billion dollars; and in fact in all recent years the net purchase by the RBI has been positive (Reserve Bank of India, 2008a).

The build up of reserves is clear from Table 1, and the accompanying graph. From 1977 to 1990 India's foreign exchange balance hovered around five billion dollars. In the early 1990s the rupee was put on a float and, from then onwards, the way for the RBI to influence the exchange rate was by buying and selling dollars. From 1993-4 the rise in foreign exchange reserves—it is believed that this is held largely in dollars—has been exponential, as can be seen with the naked eye from Graph 1.

Table 1. India's Foreign Exchange Reserves

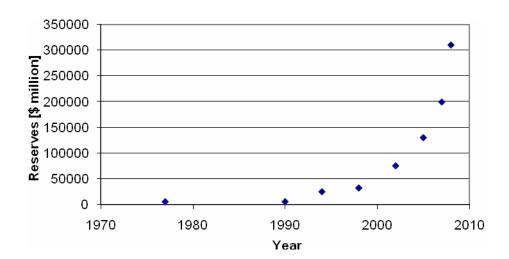
Year	\$ millions
1977	5,824
1990	5,834
1994	25,186
1998	32,490
2002	75,428
2005	130,000
2007	199,179
2008, 31 March	309,723

**Source**: Up to 2007-**Economic Surveys**, Ministry of Finance, Government of India; 2008-Reserve Bank of India (2008).

It should be clarified that the liberalization in the exchange rate policy did not happen at one go. The first Gulf War, 1990-91, precipitated a balance of payments crisis for India and this prompted policy changes and the liberalization of the exchange rate regime. Initially, in 1992, a 'dual exchange rate regime' was instituted. There was the

Report of the High Level Committee on Balance of Payments chaired by C. Rangarajan, which recommended the broad outlines of a market determined exchange rate regime. Current account convertibility was instituted in 1994, and a legal framework to assure such convertibility was put into place in June 2000 (see Reddy, 2002, for an account of this regime change)<sup>3</sup>. India does not have full capital account convertibility, though capital account conversions are permitted on a case-by-case basis.

Graph 1



The huge build up of foreign exchange reserves in developing nations is by no means confined to India and China. The next table lists the foreign exchange reserves of the top ten reserve-holding nations in 2007. The list includes China, with 1.5 trillion dollars, India, Brazil, Algeria, South Korea and Malaysia. In 1990, none of these nations, apart from China, figured in the top ten list. And over these 17 years China moved up from being 9<sup>th</sup> to 1<sup>st</sup> in terms of aggregate holding. This is so with Hongkong's reserves being treated as separate from that of Mainland China's. For most industrialized nations, foreign currency reserves have risen or declined relatively marginally. The change in rankings has been caused mainly by the steep rise in reserves of the developing nations. These nations that have accumulated a lot of reserves are also nations that have done very well in terms of exports—for India this is a recent phenomenon. Whether this has happened by design or the compulsions of political economy, this does suggest a

\_

<sup>&</sup>lt;sup>3</sup> For a detailed account of the exchange rate regimes before 1991, see Jalan (1992).

tendency on the part of central banks to maintain a depreciated exchange rate. Some of this could be a consequence of capital flows into these nations—India got record flows of dollars into its bourses—but a part of this was arguably achieved through an exchange rate float that was managed so as to make the nation's exports attractive.

Table 2: Foreign Exchange Reserve (US\$, billions) - Top 10 countries in 2007

	1990	2007
China,P.R.: Mainland	28.6	1,528.3
Japan	69.5	948.4
Russia	n.a.	464.0
India	5.8	266.5
Korea	14.5	261.8
Euro Area	n.a.	203.5
Brazil	7.4	179.4
Singapore	27.6	162.5
China,P.R.:Hong Kong	24.6	152.6
Algeria	0.7	110.2
Malaysia	9.3	100.6

**Source**: IMF **International Financial Statistics**. For India, 1990: Ministry of Finance, Government of India

This does not mean that these currencies are immune to attacks that bring down their value further. The Asian crisis of 1997 came when, by the above criteria, the exchange rates in the relevant Asian nations were low. But for reasons not relevant in the present context and too complicated to go into here, the currencies were nevertheless vulnerable to attack and rapid devaluation<sup>4</sup>.

The present paper is not concerned with the rationale behind a central bank's aim, but in the pure mechanics of how it goes about achieving its objective. Suppose, for whatever be the reason, the RBI wants to devalue the rupee *vis-a-vis* the dollar. Since

\_

<sup>&</sup>lt;sup>4</sup> I discuss some of this Basu (2003).

India is on a floating exchange rate system, where banks and other foreign exchange dealers are free to announce the exchange rate (or, equivalently, the rupee price of the dollar), the RBI cannot influence the rate by diktat but by buying and selling on the foreign exchange market. It is believed that the way the RBI devalues the rupee is to ask a public-sector bank to buy dollars from the market. This typically raises the price of dollars and so, equivalently, causes the rupee to depreciate. Usually, the RBI stays behind the scene and the only visible action on the market is that of a public sector bank making a large purchase of dollars. Here is **Mint** newspaper's web edition, **Livemint.com**, August 20, 2008 (2:45 pm), speculating about central bank intervention in India: "State-run Indian banks were seen selling dollars to help the rupee recover from a 17-month low [...]. India's central bank uses state-run banks to intervene if it wants to slow a rupee decline or prevent it from rising too quickly, and private bank dealers said Wednesday's dollar selling looked like intervention."

This is by no means special to the Indian central bank. To quote from a textbook (Auerbach, 1982, p. 414): "This method of influencing exchange rates is not always easy to detect. The central bank may have parties in the private sector intervene for them." In the U.S., to effect an intervention in the foreign exchange market, the Fed will often contact a dealing bank, such as Citibank and buy currency at Citibank's quoted rate (Lyons, 2001). Moreover, a lot of the Fed's interventions, by some counts nearly half of them, are done secretly (Hung, 1997). And, often the explicit purpose of the Fed's intervention is to influence the exchange rate (Evans and Lyons, 2000).

This method has its share of practical problems. There is, for instance, the risk of front-running. That is, the bank that executes the RBI's order knows that this will raise the price of the dollar and so can buy some dollars for itself first, before executing the RBI's order, and then, after the value of the dollar rises, sell off its own dollars, thereby making a quick (and illegal) profit. In the present paper I shall, however, rule out such possibilities and assume that the central bank's order is carried out by the public sector bank with no effort to make use of insider information. Hence, *in the model*, the RBI and the public sector bank that does its bidding may be treated as the same agent.

The question that I am interested in is the modality of intervention. If the government's interest is in devaluing the rupee (and the build up of dollar reserves is an

unintended consequence of this) then is placing an order to buy up dollars the best way to achieve this? Are there not other ways of intervening in the foreign exchange market whereby one gets a bigger bang for the buck, that is, devalue the rupee more with this same total purchase of dollars? The answer is yes and the main aim of this paper is to demonstrate this. Let us here call the current mode of intervention by the RBI a 'quantity intervention' whereby the RBI simply states the quantity of dollars that it wants the public sector bank to buy on its behalf. There seems to have been very little research on whether this is the best mode of intervention<sup>5</sup>.

This paper questions if quantity interventions (buying or selling dollars) is the best kind of intervention. It is argued that there are other forms of intervention that may be superior. In particular, the paper develops the idea of what will be called a 'schedule intervention' and argues that central banks interested in influencing the nation's exchange rate, be it the RBI in India or the PBC in China, should use schedule interventions. Broadly speaking a schedule intervention is one where the central bank or its agent bank enters the foreign exchange market not with a fixed quantity demand but with a demand that is conditional on price.

If the foreign exchange market is fully competitive, then there is no advantage to be had from a schedule intervention, but if the market has big banks and dealers who are strategic agents, as I believe there are in the Indian market, then schedule interventions can be vastly more effective. There is little work on the microfoundations of central bank intervention in developing countries, and I view the present paper as no more than a first step.

The paper is meant to be a small contribution to the general theory of micromarket structure. While for shares and stocks there is a lot of theoretical work (see O'Hara, 1995), the theoretical study of the market microstructure remains quite inadequate. A variety of topics deserve attention, from the formation of the spread to the actual institutional structure of trade (Bhanumurthy, 2008). The present paper has no such general ambition but addresses a problem that is of concern to central banks in

\_

<sup>&</sup>lt;sup>5</sup> In the context of industrialized nations there is work on how various microeconomic details of the intervention can have different impacts. For instance, the timing of the intervention matters and so does the ability to coordinate the intervention across multiple central banks (Dominguez and Frankel, 1993; Lyons, 2001).

developing nations that have a floating exchange rate but at the same time perceive the need to occasionally intervene to keep it 'on course'. The paper makes a precise policy recommendation by proposing a switchover to schedule interventions by central banks.

#### 3. A Sketch of the Problem

Let me begin by outlining, in purely intuitive terms, what the problem is. Suppose there are two currencies, the domestic one, henceforth, rupees, and the foreign one, dollars. Let the demand curve for dollars be described by the line AB in Figure 1 and the supply curve by the upward sloping line. If this were a competitive market the equilibrium exchange rate or, equivalently, the price of dollars would be p\*, as shown.

Now suppose, for whatever reason, the central bank wants to devalue the currency to the exchange rate p\*\*. If this is to be done not by law or diktat but by market intervention, a natural way to achieve this is for the central bank to demand CD dollars. This 'quantity intervention' would push the demand curve out to A'B' and raise the price of dollars to p\*\*. This, in a nutshell, is what India's RBI and legions of central banks in developing countries do. Note that in the process the central bank would end up acquiring CD dollars and releasing CD multiplied by p\*\* rupees onto the market, thereby raising tricky questions of inflationary pressures and the need to sterilize. That this is a natural way of thinking about how to influence exchange rates is clear from textbook descriptions of what central banks do under 'managed' or 'dirty' float. "[The method whereby] the central banks step in and buy and sell currencies to prevent them from falling or rising in value beyond predetermined limits have also been used." (Auerbach, 1982, p. 414).

In a competitive market of this kind, there is no advantage to an intervention where the extent of demand for dollars is made contingent on the price. As long as the new demand curve goes through point D the net effect is the same. If, for instance, the central bank decides to buy less dollars if the price is low so that the new aggregate demand curve is given by the broken line in Figure 1, which goes through D, the final

9

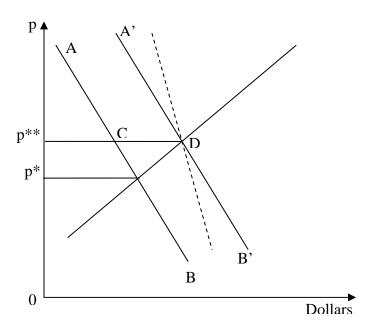
\_

<sup>&</sup>lt;sup>6</sup> It is possible to think of a case where the price of the dollar was originally at p\*\* but market demand fell temporarily and the price fell to p\*; the central bank simply wants to stabilize this fluctuation.

equilibrium is still at price p\*\* and the amount of dollars acquired by the central bank is still CD.

At first sight this seems natural enough. If the demand for dollars is the same at the equilibrium price, in this case p\*\*, then the fact that demand would be different at out-of-equilibrium prices can surely not influence the equilibrium price. This logic, however, is true only for purely competitive markets.

Figure 1



If we allow for the existence of large firms which are strategic agents, quantity interventions by the central bank works as above—this is demonstrated in section 5—but now there emerges other forms of interventions, which entail different kinds of behavior by the central bank at out-of-equilibrium prices, which can cause the equilibrium to change<sup>7</sup>. Moreover, such interventions are more efficient from the point of view of the bank. This is demonstrated in section 6. Our first task however is to formally describe the

\_

<sup>&</sup>lt;sup>7</sup> In extensive-form game theory this is standard wisdom that out of equilibrium plans can affect what the equilibrium will be. Similar results have also been seen in trade theory. It is, for instance, known that having an exchange rate band set by the central bank, can influence the behavior of the exchange rate even *within* the band (Krugman, 1991; Helpman, Liederman, Bufman, 1994).

structure of the foreign exchange market in which there are small (that is, price-taking) and large (that is, oligopsonistic or oligopolistic) firms.

#### 4. The Market Structure

Before analyzing central bank interventions it is necessary to describe the structure of forex markets. I shall here describe a very simple structure which has just enough complexity to analyze the kind of intervention that India's RBI uses in the forex market, and allows me to explain clearly the kind of policy that I am recommending.

As before, there is only one foreign currency, the dollar, and the domestic currency is the rupee. Let p be the rupee price of each dollar. Hence, a rise in p amounts to a devaluation of the rupee, or a depreciation of the currency.

Let the supply function of dollars in India be given by

$$s = s(p)$$

where s'(p) > 0. That is, if the price of dollar in India rises, more dollars come into the country.

Since the intervention will be studied here from the demand side, we need to characterize demand in a more elaborate fashion. It will be assumed that three kinds of agents demand dollars. First, there is the price-taking fringe. These may be ordinary citizens who or small foreign exchange bureaus that cannot affect the exchange rate, p. They may also be small firms such as in Dixit (1989) who take the exchange rate as given and do their calculations based on that. Let the aggregate demand for dollars from this class of agents be given by:

$$d = d(p)$$

As usual, we assume d'(p) < 0.

Second, there are  $n \ge 2$  large foreign exchange dealers. These are strategic agents who can affect the exchange rate by their acts of buying and selling dollars. In the case of India, these are, typically, the members of the Foreign Exchange Dealers Association of India (FEDAI), which currently has 89 members, consisting of many large banks and other financial institutions. I shall refer to these firms, henceforth, as 'forex

dealers' or simply 'dealers'. It will be assumed that the dealers have some special use for dollars, not available to individual citizens and the small exchange bureaus. In particular, each of these n agents reaps a value of u for each dollar that it purchases from the domestic foreign exchange market. I could have allowed for idiosyncratic variations in the value of u. Since this would add nothing but algebraic complexity, I prefer to assume all dealers have the same u. If  $\tilde{p}$  is the price where  $d(\tilde{p}) = s(\tilde{p})$ , then we assume  $u > \tilde{p}$ . This ensures that it is worthwhile for these dealers to buy dollars on the domestic foreign exchange market. Hence, the market structure that I am considering is an oligopsony, along with a price-taking fringe of buyers and sellers. The importance of a few big dealers that trade large volumes in the Indian foreign exchange markets seems undeniable (Bhanumurthy, 2008).

While in my model there are large demanders of dollars, there are no large sellers. This is purely for analytical convenience, since it allows me to use some standard results from industrial organization theory. Once we have strategic agents on both sides of the market, we need to develop somewhat more elaborate methods of analysis. However, all my main arguments remain intact in such a more general model. This is demonstrated in the closing section of this paper, which may be viewed as a technical appendix.

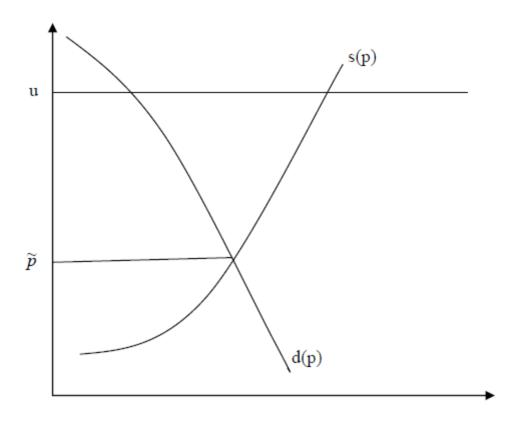
Finally, there are a few public sector banks that come into this market only when directed to do so by the central bank, the RBI.<sup>8</sup> In this and the next section, we could think of the RBI and the public sector banks that carry out its orders as the same agent. The parameters of the market described above are represented graphically in Figure 2.

Let me first characterize the equilibrium exchange rate with no RBI intervention. Hence, to start with, we can ignore the public sector banks. The market structure that I am proposing is the one used by Encaoua and Jacquemin (1980) (see also Dixit and Stern, 1982; Basu, 1993). The equilibrium in this market is the Nash equilibrium among the n oligopsonists, keeping in mind that the price-taking fringe will respond non-strategically to whatever price comes into existence.

-

<sup>&</sup>lt;sup>8</sup> In India most large public sectors banks are members of FEDAI. That is they participate in the forex market as regular players and also do the RBI's bidding. Purely for simplicity I shall here think of these banks as purely agents of the RBI. This is easy to generalize.

Figure 2



Suppose the dealer i, demands  $x_i$  dollars, for all i. Then the demand for dollars will be equal to supply of dollars if and only if the exchange rate, p, is such that the following condition is true:

$$x_1 + \dots + x_n + d(p) = s(p)$$
 (1)

or

$$x_1 + ... + x_n = s(p) - d(p) \equiv \psi(p)$$
 (2)

We shall henceforth refer to  $\psi$  as the 'net supply function,' and it will be assumed that a p satisfying equation (2) exists. This can be achieved by placing a domain restriction on the values that  $x_i$ 's can take or, more directly, by assuming that for every number z, there exists a p such that  $\psi(p) = z$ . Since s'(p) > 0, d'(p) < 0, for all p,  $\psi'(p) > 0$ , for all p.

Hence, the inverse of  $\psi$  exists. Let  $\phi$  denote the inverse. So  $\phi$  is the 'inverse net supply function'.

Hence, if the *n* large forex dealers demand  $x_1,...,x_n$  dollars, the price of dollars or the exchange rate will be given by

$$p = \phi(x_1 + ... + x_n) \tag{3}$$

And, in that case, the profit,  $\pi_i$ , earned by the forex dealer, i, is given by

$$\pi_i(x_1, ..., x_n) = ux_i - x_i \phi(x_1 + ... + x_n)$$
(4)

Therefore,  $x^* = (x_1^*, ..., x_n^*)$  is an *equilibrium* if and only if  $x^*$  is a Nash equilibrium of the *n*-player game in which each player's payoff function is given by (4).

We shall say that  $p^*$  is the *equilibrium exchange rate* if and only if  $p^* = \phi(x_1^* + ... + x_n^*)$ , where  $x^*$  is an equilibrium.

# 5. Quantity Intervention

We are now in a position to study how a quantity intervention by the RBI affects the exchange rate. As was explained in the introduction, if the RBI wants to depreciate the rupee it asks a public sector bank to buy dollars on the forex market. Let us check out how this works in the model.

Suppose the RBI asks a public sector bank to enter the forex market and buy D dollars. The public sector bank mechanically executes the RBI's order. As soon as it does this, (1) has to be rewritten as:

$$x_1 + ... + x_n + d(p) + D = s(p)$$
 (5)

since, in addition to the forex dealers and the price-taking fringe, there is now demand D from a public sector bank. So a price p that equates demand and supply has to take this into account.

Hence, if the public-sector bank demands D dollars and the n large forex dealers demand  $x = (x_1, ..., x_n)$ , using the notation in equation (2), the exchange rate, p, will be given by

$$x_1 + ... + x_n + D = \psi(p)$$

$$p = \phi(x_1 + ... + x_n + D).$$

Firm *i's* profit is now given by

$$\pi_i(x_1, ..., x_n) = ux_i - x_i \phi(x_1 + ... + x_n + D). \tag{6}$$

Since the equilibrium in this will depend on D, I shall denote the equilibrium by  $(x_1(D),...,x_n(D))$ , where this vector constitutes a Nash equilibrium of the above game. The equilibrium exchange rate p(D) is then given by

$$p(D) = \phi(x_1(D) + \dots + x_n(D) + D) \tag{7}$$

It is easy to see that the model described in the previous section is a special case of this model, where D = 0. Thus the equilibrium exchange rate,  $p^*$ , in that model is nothing but p(0).

Let us begin with an equilibrium with no central bank intervention. Hence D = 0 and the equilibrium exchange rate is given by p(0). We want to study what happens when D is raised to some positive number, that is, to compare p(0) with p(D), where D > 0. It is easy to establish that p(D) > p(0). To do so – and this is anyway useful for what I do later – it is useful to characterize the equilibrium in the above model.

Since dealer *i* chooses  $x_i$  to maximize  $\pi_i$ , described in (6), we have the following first-order condition.

$$u = \phi(x_1 + \dots + x_n + D) + x_i \phi'(x_1 + \dots + x_n + D)$$
 (8)

It will be assumed that (8) has a unique solution for  $x_i$ . It is easy to place restrictions on the primitives of the model to ensure this.

It is straightforward to check that in equilibrium all dealers will choose the same quantity. Hence,

$$x_1 = x_2 = \dots = x_n \equiv x(D).$$

In other words each dealer demands the same amount of dollars in equilibrium, which is denoted by x(D).

Therefore, the equilibrium exchange rate is given by

$$p(D) = \phi(nx(D) + D) \tag{9}$$

Our task now is to check how p changes in response to changes in D.

As is familiar from standard industrial organization theory, the answer depends on the strategic relationship between the forex dealers. Let us assume that there is strategic substitutability among the dealers. This is the natural assumption for firms dealing in the same product and a wide class of demand and supply functions  $d(\cdot)$  and  $s(\cdot)$ , for instance, when these are linear Bulow, Geanakoplos and Klemperer, 1985; Singh and Vives, 1984).

Strategic substitutability means that for every firm i, if another firm j increases  $x_j$ , firm i will prefer to decrease  $x_i$ . If we differentiate through equation (8) with respect to  $x_j$ , and re-organize the equation, the strategic substitutability condition reduces to

$$\frac{dx_i}{dx_j} = \frac{-\phi'(\cdot)}{\phi'(\cdot) + x_i \phi''(\cdot)} < 0$$

Since  $\phi$  ' > 0, this condition reduces to:

$$\phi'(\cdot) + x_i \phi''(\cdot) > 0 \tag{10}$$

From now on I shall assume this to be true. It will be interesting to see however that for the main policy prescription that will emerge from this paper this condition will not be needed.

The consequence of a state-owned bank entering the forex market to buy dollars is now easy to see.

Note that in equilibrium condition (8) reduces to:

$$u = \phi(nx(D) + D) + x(D)\phi'(nx(D) + D)$$

Using y(D) to denote the aggregate demand for dollars, this can be rewritten as:

$$u = \phi(y(D)) + (\frac{y(D) - D}{n})\phi'(y(D)). \tag{11}$$

Differentiating through with respect to D and rearranging terms we get

$$\frac{dy(D)}{dD} = \frac{\phi'(y(D))}{n[\phi'(y(D)) + x(D)\phi''(y(D))]}$$

Strategic substitutability, namely, condition (10), implies that  $\frac{dy(D)}{dD} > 0$ . Since  $\phi' > 0$ , it follows that as D increases, p rises, that is, the rupee loses value or, equivalently, there is devaluation.

This is precisely the justification behind the common practice of depreciating the domestic currency by asking state-owned firms to buy up foreign currency on the domestic forex market. This is what the RBI does in India. It is believed that the RBI uses one or more public sector banks to do this. It is easy to see that whether one bank is asked to buy D dollars or m banks are asked to buy  $D_1,...,D_m$  dollars, where  $D_1 + ... + D_m = D$ , makes no difference to the ultimate impact on the exchange rate. So it may be fine for the central bank to be cavalier about whether to involve one or several public sector banks when engaging in quantity interventions. There can be other kinds of interventions where the efficacy of the intervention depends critically on the number of agents used by the central bank. This will come up only in the next section.

In summary, we now have a clear micro foundation for standard central bank interventions. Suppose that the exchange rate that prevails with no central bank intervention, p(0), is considered too low by the central bank. The central bank would then direct one or more state-owned banks to purchase a total of D dollars. This will result in a new equilibrium exchange rate p(D). As we just saw p(D) > p(0). So the objective of the central bank is achieved, at least in some measure.

However, with this intervention the central bank picks up a reserve of D dollars and injects p(D)D rupees into the economy. If, for whatever reason, a government decides to make permanent corrections to the exchange rate by, for instance, keeping the currency depreciated over long stretches of time, the reserves can rise way above what the nation could conceivably need. The bloated reserves in such cases are simply the unintended consequence of keeping the exchange rate devalued. As we know, this reserve build up can have undesirable implications. For one, the domestic currency released on the market in order to buy up the dollars often generates inflationary pressures. Governments then engage in difficult sterilization interventions to dampen inflation.

This raises the following question: Can we not conceive of other ways of intervening in the foreign exchange market which influences the exchange rate without

building up too much reserve? To put it differently, if a government decides to buy D dollars, are there other ways of executing this purchase whereby one gets a 'bigger bang for the buck', that is, a larger impact on the exchange rate. The answer is yes. The next section discusses this and recommends a particular form of intervention called schedule intervention to central banks like those of India, China and South Korea, which seem to face the problem of 'reserves glut'.

#### 6. Schedule Intervention

There are indeed many other ways in which a central bank can try to boost demand, some more effective than others, with different kinds of unintended effects associated with each intervention. There has been little research on this subject. Since we are analyzing policy in an area where billions of dollars are involved, a small alteration in the mechanism for intervention can have an enormous aggregate consequence. It is a bit like auctions, where theoretical research has yielded large, actual gains.

For central bank intervention, an alternative form of action is to go for strategic *price* intervention. Suppose, as before, the RBI wants to devalue the rupee. It could call a public-sector bank and offer the bank u rupees for every dollar, where the u here is the same u that the large forex dealers earn as revenue from each dollar that they acquire. In other words, the central bank puts a public sector bank on par with a forex dealer. What this simply means is that the forex market functions exactly like before but with n+1 strategic agents – the n dealers and 1 bank which now plays like a dealer. It follows from standard industrial organization theory that this will raise p.

For this kind of intervention, unlike a 'quantity intervention' it matters a lot whether the RBI deputes one or many banks to buy dollars for it at a value of u. Suppose it delegates this task to m banks. This will mean that the foreign market will have n+m strategic agents. We know from standard oligopsony theory that as n+m increases, the market price for dollars, p, will increase. In fact, as n+m goes to infinity, the market price will go to u.

Depending on the government's objectives, it should consider these alternative mechanisms for intervention. All the interventions discussed thus far will result in the accumulation of foreign exchange reserves in the hand of the central bank, thereby releasing more domestic currency into the economy.

If the RBI's aim is to influence the exchange rate with minimal effect on reserves then none of the above interventions is as good as what I shall call a 'schedule intervention'? This entails the RBI calling up a public sector bank and quoting neither a fixed quantity, D, nor a price, u, but a *function*, or a *schedule*, f, which converts every possible market price, p, to a quantity of dollars, f(p). When we say that the RBI offers a public sector bank a function, f, what we mean is that, if the market price is p, the RBI will want the public sector bank to buy f(p) dollars from the market. In other words, the bank enters the market not with the aim of a fixed quantity it will buy no matter what the price, but a whole 'schedule' of plans<sup>9</sup>. Of course, the function f can take the form f(p) = D, for all p. In other words, a quantity intervention is a special case of a schedule intervention.

Let us now analyze how the equilibrium exchange gets determined in a foreign exchange market which has all the same features as the model in Sections 4 and 5 with one alteration: The (public sector) bank enters the market not with a fixed demand for dollars but a price-dependent schedule of demand described by the function f. The dealers take this into account and respond in their own interest f0.

It is obvious that a price p will cause demand and supply to be equal if, instead of (5), we have the following:

$$x_1 + \dots + x_n + d(p) + f(p) = s(p).$$
 (12)

<sup>&</sup>lt;sup>9</sup> I am here modeling the entire interaction as a simultaneous game. Since, the central bank effectively uses a 'reaction function' as strategy, the outcome would be unchanged if it was assumed that it was the first mover after whom all the dealers simultaneously made their choice. The game would then be like an oligopoly with the central bank having the advantage of a first mover, for instance, in an entry-deterrence

model (Dixit, 1980). But in this case this would make no difference to our results.

<sup>&</sup>lt;sup>10</sup> The idea of an agent entering an oligopoly with a function or schedule as opposed to a quantity or price is an idea that had received some attention in the 1980s (see, for instance, Bresnahan, 1981; Klemperer and Meyer, 1988, 1989).

The price, p, that solves this equation can be written as a function of the total demand of the dealers and intervention function, f (since d and s remain unchanged, these may be ignored). Hence<sup>11</sup>,

$$p = \hat{\phi}(x_1 + \dots + x_n, f) \tag{13}$$

The payoff function of dealer i, is now given by:

$$\pi_i(x, f) = ux_i - \hat{\phi}(x_1 + \dots + x_n, f)x_i \tag{14}$$

An equilibrium,  $x^*$ , is a Nash equilibrium of this game and an equilibrium exchange rate is  $p^* = \hat{\phi}(x_1^* + ... + x_n^*, f)$ , where  $x^*$  is an equilibrium.

The central bank's problem is to influence  $p^*$  by choosing an appropriate schedule intervention, f. It can be shown that the central bank can now depreciate the currency with no build up of foreign exchange reserves.

To explain this, let me first define no intervention to be a schedule intervention,  $f^0$  such that, for all p,  $f^0(p) = 0$ . Define

$$p^{0} \equiv \hat{\phi}(x_{1}^{*} + ... + x_{n}^{*}, f^{0})$$

where  $x^*$  is a Nash equilibrium. Clearly  $p^0$  is the equilibrium exchange rate of Section 4 or the equilibrium exchange rate p(0) of Section 5.

Next, define  $\tilde{p}$  to be such that  $s(\tilde{p}) = d(\tilde{p})$ .

**Proposition:** For any price  $\hat{p} \in (\tilde{p}, u)$ , there exists a schedule intervention, f, such that the equilibrium exchange rate moves to  $\hat{p}$  and the net purchase of dollars by the central bank is zero.

I shall here prove this by actually constructing a schedule intervention, f, such that the entry of a bank into the foreign exchange market with demand described by f achieves what the theorem above describes.

<sup>&</sup>lt;sup>11</sup> Strictly, this can be multi-valued and so be a correspondence. However, the particular f function that we will ultimately use will ensure that  $\hat{\phi}$  is a function; and so it is harmless to treat this as a function from the start.

Consider a  $\hat{p} \in (\tilde{p}, u)$ . Most readers may wish to consider  $\hat{p} \in (p^0, u)$  since that will mean that the intervention seeks to devalue the currency. Since much of the paper has been concerned with devaluation, most readers will find this to be the relevant case though the proof is unchanged whether we are interested in deflating or inflating the currency.

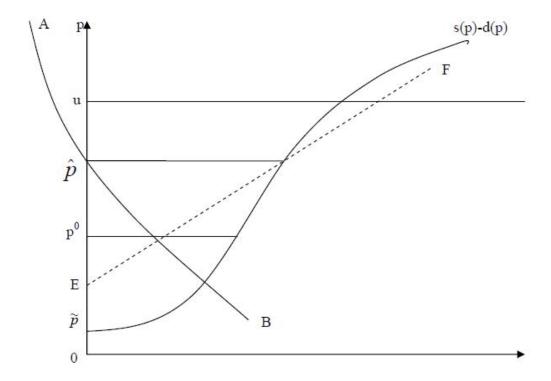
Seeking the reader's indulgence for the unseemly size of the intervention function, here it is:

$$f(p) = s(p) - d(p) - \left[ \frac{s(\hat{p}) - d(\hat{p})}{n(u - \hat{p})} \right] p - \left[ \frac{s(\hat{p}) - d(\hat{p})}{n(u - \hat{p})} \right] [n(u - \hat{p}) - \hat{p}]$$
(15)

Before showing that this intervention function does the job, it may be useful for the reader to carry a pictorial representation of what is being done. This is represented in Figure 3. The line going north-east from  $\tilde{p}$  is the net supply curve, s(p) - d(p), that is, the excess supply over and above what is demanded by the price-taking fringe. This is the effective supply function of dollars to the large dealers and the state-owned bank. When the state bank is absent, let  $p^0$  be the equilibrium exchange rate. We are looking for a schedule intervention function, f, such that a state bank entering the foreign exchange market with that demand function drives the market price to  $\hat{p}$  (for some arbitrarily chosen point,  $\hat{p}$ , between u and  $\tilde{p}$ ); and the market clears with zero demand (in equilibrium) from the state bank.

My claim is that the f function described in (15) will do it. It is easy to see that f(p) < 0 for all  $p > \hat{p}$  and f(p) > 0 for all  $p < \hat{p}$ . Hence, the graph of f looks like the line AB. (15) was constructed so that s(p) - d(p) - f(p) is a straight line. This is shown by line EF, which goes through the point  $(s(\hat{p}) - d(\hat{p}), \hat{p})$ .

Figure 3



To complete the proof, insert the f(p) function (15) in (12). This gives us

$$x_1 + \dots + x_n = \left[ \frac{s(\hat{p}) - d(\hat{p})}{n(u - \hat{p})} \right] [p + n(u - \hat{p}) - \hat{p}]$$
 (16)

$$p = [x_1 + \dots + x_n] \frac{n(u - \hat{p})}{[s(\hat{p}) - d(\hat{p})]} - [n(u - \hat{p}) - \hat{p}]$$
(17)

Using (14) we know that for every dealer, i,

$$\pi_i = ux_i - px_i,$$

where p is as defined by (17).

By writing each firm's first order conditions and solving the n equations, it is easy to check that each dealer will demand  $\hat{x}$  as defined below.

$$\hat{x} = [s(\hat{p}) - d(\hat{p})]/n$$
 (18)

Hence, total demand by the dealers is  $n\hat{x} = s(\hat{p}) - d(\hat{p})$ . It is obvious from (15) that  $f(\hat{p}) = 0$ . Hence,  $\hat{p}$  is the equilibrium exchange rate. And the state-owned firm buys no

dollars and sells no dollars at the equilibrium. The planned devaluation is achieved with no build up or erosion of resources.

The formal analysis may create the impression that the build up of resources with the central bank is always undesirable. That is of course not so. As the former governor of India's central bank, Y. V. Reddy (2008, p.1106) observed, while "excess" reserves can be costly in terms of foregone income and have other "fiscal costs", large reserves "enhances the confidence in the economy, particularly of the emerging market economies and results in a better sovereign rating" and it also "enhances the capacity to absorb shocks." Interestingly, this does not detract from the advantages of a schedule intervention. It is easy to see that one can always alter the nature of the *f* function to change the amount of foreign exchange purchased by the central bank in equilibrium. While the schedule intervention (15) results in zero reserve accumulation, the central bank does not *have* to use this particular schedule. The advantage of using a schedule intervention is that it allows the central bank to separate out the objectives of exchange rate adjustment and reserve accumulation. It can set separate targets for both these and find a suitable schedule intervention that delivers on both.

## 7. Reality Check

The intervention technique suggested in this paper, in principle, causes no build up of foreign exchange and so this can be of use to the governments and central banks of China, India, South Korea, which may have an interest in boosting exports by keeping their currency undervalued, but are wary of the build up of reserves that this causes. This policy recommendation should however come with some warnings of distortions that are

.

<sup>&</sup>lt;sup>12</sup> This is related to a larger methodological point. Economists tend to evaluate exchange rate regimes (and, by the same logic, exchange rate levels in a managed float regime) by standard welfare criteria of measuring economic costs and benefits and then picking the best or at least a Pareto efficient one. This approach has its limitations since regimes and exchange rate levels can affect different groups differently and raise distributional questions and this is turn raises issues of political economy. This is well-known (see for instance, Helpman and Liederman (1991). But, as this quote shows, even beyond that, there are issues of psychology and trader and consumer confidence that can prompt a central bank to have preferences over exchange regimes, exchange rates and foreign exchange reserve levels, which are not captured by standard economics or even standard political economy considerations.

bound to occur because reality does not function as smoothly as a Cournot-type world as described in this paper.

In most of our models – a la Cournot or Walras--trade does not occur till we get to the equilibrium. In reality after the central bank intervenes via the state bank, the buying and selling of dollars will occur as the exchange rate gradually finds it way to the equilibrium  $\hat{p}$ . During this period of out-of-equilibrium adjustment, the state bank (and therefore the central bank) is bound to accumulate some forex reserves. If the state bank tries not to make any purchases till the exchange rate settles into an equilibrium this will raise questions of credibility concerning its declared aim of making different kinds of interventions at various possible prices. Hence, the state bank has to act even when prices are evidently moving around in search of an equilibrium. One would, however, expect the accumulation from this to be much less than in the traditional forms of intervention which entail accumulating dollars even in equilibrium.

A second source of error must arise from the difficulty of determining the actual form of the schedule function f described in (15). This will basically require a knowledge of the demand and supply functions of foreign exchange of the small, price-taking agents, an exercise that is likely to require a lot of empirical work. This will not be easy but, within a margin of error, not that hard either. Most central banks collect copious amounts of data; and though they do not release these to outside researchers, they have enough inhouse economists and econometricians to estimate these functions. Moreover, even for the standard quantity intervention, the central bank will need to estimate the market's demand and supply at the targeted price in order to calculate what its own demand, D, should be.

Even if one does not have a sense of the precise demand and supply functions one can develop some simple rules of thumb that try to approximate (15). Thus if the central bank wants to depreciate its currency it should choose an f function such that at the existing exchange rate,  $p^0$ ,  $f(p^0) > 0$ ; and one can set f'(p) to be positive but a low number just to be on the safe side.

Further, the standard quantity intervention looks simple but is actually quite demanding. Its success demands critically on there being strategic substitutability among the actions of all the forex dealers. In the absence of this, an additional demand for

dollars can actually drive down the price of dollars and cause the exchange rate to appreciate. The schedule intervention being prescribed in this paper is independent of strategic substitutability or complementarity.

The main motivation for this paper is, however, more than to prescribe a particular structure of central bank intervention. The aim is to demonstrate that the micro mechanism of intervention is well worth studying. This has largely escaped the attention of macro and monetary economists but large efficiency gains are possible through research in this field and the development of a finer set of policy tools for central bank intervention.

#### 8. Technical Appendix: Bilateral Oligopoly and Schedule Intervention

The aim of this technical section is to demonstrate that the policy instrument developed above would continue to work if there were large, strategic foreign-exchange dealers both on the buying and selling side of the market. This, however, requires us to develop a model of bilateral oligopoly. Fortunately, there is a natural way to extend the above model to achieve this.

Consider two sets of strategic agents or foreign exchange dealers. Those who, as in the above sections, can realize u rupees worth of value from each dollar they buy on the Indian market, and those who have a source of cheap dollars.--They are able to obtain dollars at the rate of Rs. r for each dollar. They can then sell these dollars on the domestic market. The latter will be called 'selling dealers' and the former 'buying dealers'. We shall assume that there are n buying dealers and m selling dealers.

Of course, u > r. As before, the price-taking agents have a demand and supply of dollars given by d(p) and s(p). This market with price-taking agents clears at a price that lies between u and r. That is, if  $d(\tilde{p}) = s(\tilde{p})$ , then  $\tilde{p} \in (r, u)$ .

If the *n* buying dealers demand  $x_1,...,x_n$  units of dollars and the selling dealers wish to sell  $y_1,...,y_m$  dollars, then (assuming, as before, free market behavior) the price of dollars, *p*, will be given by the following.

$$\sum x_i - \sum y_i + d(p) = s(p) \tag{19}$$

Hence, using the same notation as above we have:

$$\psi(p) = \sum x_i - \sum y_i$$
$$p = \phi(\sum x_i - \sum y_i)$$

or

It follows that the buying and selling dealers' profit functions are given by, respectively

$$\pi_{j}^{B}(x_{1},...,x_{n},y_{1},...,y_{m}) = ux_{j} - \phi(\sum x_{i} - \sum y_{i})x_{j}, \quad j = 1,...,n$$

$$\pi_{j}^{S}(x_{1},...,x_{n},y_{1},...,y_{m}) = \phi(\sum x_{i} - \sum y_{i})y_{j} - ry_{j}, \quad j = 1,...,m.$$

This gives us the following n + m first-order conditions.

$$u = \phi(\sum x_i - \sum y_i) + x_j \phi'(\sum x_i - \sum y_i), \quad j = 1, ..., n$$
  
$$r = \phi(\sum x_i - \sum y_i) - y_i \phi(\sum x_i - \sum y_i), \quad j = 1, ..., m$$

It is now easy to characterize the Nash equilibrium of this bilateral oligopoly. A demand, x, by each buying dealer and a supply, y, by each selling dealer is a Nash equilibrium or an 'equilibrium of the bilateral oligopoly' if the following equations hold.

$$u = \phi(nx - my) + x\phi'(nx - my) \tag{20}$$

$$r = \phi(nx - my) - y\phi'(nx - my)$$
(21)

The only novelty in this exercise is to have a price taking fringe. In the absence of this it is difficult to define a meaningful equilibrium for a bilateral oligopoly and one has to indulge in the complications that can be found in the literature. But as long as there are some price-taking agents in the model giving rise to an upward-sloping net supply curve, an equilibrium becomes easy to define and work with.

Some properties of the equilibrium are immediately obvious. Since  $\phi'>0$ , it follows that the exchange rate or equilibrium price of dollars,  $\phi(nx-my)$ , will always be greater than r and less than u. It is also possible to show with a little bit of algebra that, as n goes to infinity, the exchange rate goes towards u and, as m goes to infinity, the exchange rate goes towards r. These are variants of the standard result in oligopoly theory of the convergence of the Cournot outcome to the competitive outcome, as the number of firms goes to infinity.

By an argument very similar to the one used above, it is now possible to demonstrate that for any  $\tilde{p} \in (r,u)$  it is possible for the central bank to make a schedule intervention such that the equilibrium exchange rate rises to  $\tilde{p}$  and the central bank does

not have to buy or sell dollars in equilibrium. It is the assurance of doing so at out-of-equilibrium prices that drives the large dealers to behave in such a way that the desired exchange rate is reached.

The essential technique is to recognize that, once the central bank intervenes with the schedule, f(p), equation (19) becomes

$$\sum x_{i} - \sum y_{i} + d(p) + f(p) = s(p)$$
 (22)

One has to then carry this 'correction' through the subsequent equations and then find a schedule intervention, f, in the spirit of equation (15), which directs the equilibrium price to  $\tilde{p}$ .

#### References

- Auerbach, R. D. (1982), **Money, Banking and Financial Markets**, New York: Macmillan.
- Basu, K. (1993), **Lectures in Industrial Organization Theory**, Oxford: Blackwell Publishers.
- Basu, K. (2003), 'Globalization and the Politics of International Finance,' **Journal of Economic Literature**, vol. 41, 2003.
- Basu, K. and Morita, H. (2006) 'International Credit and Welfare: A Paradoxical Theorem and its Policy Implications,' **European Economic Review**, vol. 50.
- Bhanumurthy, N. R. (2008), 'Microstructures in the Indian Foreign Exchange Markets,' mimeo: Institute of Economic Growth.
- Bresnahan, T. (1981), 'Duopoly Models with Consistent Conjectures,' **American Economic Review**, vol. 71.
- Bulow, J., Geanakoplos, J. and Klemperer, P. (1985), 'Multi-market Oligopoly: Strategic Substitutes and Complements,' **Journal of Political Economy**, vol. 93.
- Dixit, A. (1980), 'The Role of Investment in Entry-Deterrence,' **Economic Journal**, vol. 90.
- Dixit, A. (1989), 'Hysteresis, import penetration, and exchange rate pass-through,'

  Quarterly Journal of Economics, vol. 104.
- Dixit, A. and Stern, N.H. (1982), 'Oligopoly and Welfare: A Unified Presentation with Applications to Trade and Development,' **European Economic Review**, vol. 19.
- Dominguez, K. and Frankel, J. (1993), 'Does Foreign-Exchange Intervention Work?' **Institute for International Economics,** Washington, D.C.
- Encaoua, A. and Jacquemin, D. (1980), 'Degree of Monopoly, Indices of Concentration and Threat of Entry,' **International Economic Review**, vol. 21.
- Evans, M. and Lyons, R. (2000), 'The Price Impact of Currency Trades: Implications for Secret Interventions,' mimeo.
- Flanders, M. J. and Helpman, E. (1978), 'On Exchange Rate Policies for a Small Country,' **Economic Journal**, vol. 88.

- Helpman, E. and Liederman, L. (1991), 'Exchange rate Systems: New Perspectives,' inZ. Eckstein (ed.), Aspects of Central bank Policy Making, Berlin: Springer-Verlag.
- Helpman, E., Liederman, L. and Bufman, G. (1994), 'A New Breed of Exchange Rate Bands: Chile, Israel, and Mexico,' **Economic Policy**, vol. 9.
- Hung, J. (1997), 'Intervention Strategies and Exchange Rate Volatility: A Noisy Trading Perspective,' **Journal of International Money and Finance**, vol. 16.
- Jalan, B. (1992), 'Balance of Payments, 1956-1991,' in B. Jalan (ed.), **The Indian Economy: Problems and Prospects**, New Delhi: Penguin.
- Klemperer, P. and Meyer, M. (1988), 'Consistent Conjectures Equilibria: A Reformulation Showing Non-Uniqueness,' **Economics Letters**, vol. 27.
- Klemperer, P. and Meyer, M. (1989), 'Supply Function Equilibria in Oligopoly under Uncertainty,' **Econometrica**, vol. 57.
- Krugman, P. R. (1991), 'Target Zones and Exchange Rate Dynamics,' **Quarterly Journal of Economics**, vol. 106.
- Lyons, R. K. (2001), **The Microstructure Approach to Exchange Rates**, Cambridge, MA: The MIT Press.
- Mark, N. C. and Sul, D. (2001), 'Nominal Exchange Rates and Monetary Fundamentals:

  Evidence from a Small post-Bretton Woods Panel,' **Journal of International Economics**, vol. 3.
- O'Hara, M. (1995), Market Microstructure Theory, Oxford: Blackwell Publishers.
- Reddy, Y. V. (2002), 'Operationalizing Capital Account Liberalization,' in Y. V. Reddy,
  Lectures on Economic and Financial Sector Reforms in India, New Delhi:
  Oxford University Press.
- Reddy, Y. V. (2008), 'Fiscal Policy and Economic reforms,' **Reserve Bank of India Bulletin**, July, vol. 57.
- Reserve Bank of India (2008), Report on Foreign Exchange Reserves, Mumbai: RBI.
- Reserve Bank of India (2008a), Annual Report 2007-08, Mumbai: RBI.
- Sarno, L. and Taylor, M. P. (2001), 'The Microstructure of the Foreign-Exchange Market: A Selective Survey of the Literature,' Princeton Studies in International Economics, No. 89.

Singh, N. and Vives, X. (1984), 'Price and Quantity Competition in a Differentiated Duopoly,' Rand Journal of Economics, vol. 15.